1. If $A = \begin{bmatrix} -1 & \frac{3}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$, and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then $I + A + A^2 + ...$ upto ∞ is equal to

   (a) $\begin{bmatrix} 1 & -3 \\ 1 & 4 \end{bmatrix}$

   (b) $\frac{2}{7}\begin{bmatrix} 1 & -3 \\ 1 & 4 \end{bmatrix}$

   (c) $\frac{2}{7}\begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$

   (d) None of these

2. For a real number $x, [x]$ denotes the greatest integer less than or equal to x. Then, the value of $[\frac{1}{2}] + [\frac{1}{2} + \frac{1}{100}] + [\frac{1}{2} + \frac{2}{100}] + ... + [\frac{1}{2} + \frac{99}{100}]$ is

   (a) 49

   (b) 50

   (c) 48

   (d) 51

3. $\lim_{n\to\infty} \frac{2^{n+1}+3^{n+1}}{2^n+3^n}$ is equal to

   (a) 3

   (b) 2

   (c) 1

   (d) 0

4. If n is an integer between 0 and 21, then the minimum value of $n! \times (21 - n)!$ is

   (a) $9! \times 12!$

   (b) $20!$

   (c) $21!$

   (d) $10! \times 11!$

5. If n is a natural number then the number of non-negative integral solutions of $x + y + z = n$ is

(a) $\frac{n(n-1)}{2}$

(b) $\frac{n(n+1)}{2}$

(c) $\frac{(n+1)(n+2)}{2}$

(d) None of these

6. If neither α nor β is a multiple of $\frac{\pi}{2}$ and the product AB of matrices $A = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$ is the null matrix, then $\alpha - \beta$ is

(a) 0

(b) a multiple of π

(c) an odd multiple of $\frac{\pi}{2}$

(d) None of these

7. If $\tan x = n\tan y, 0 < x, y < \frac{\pi}{2}$, where n is a positive real number, then the upper bound of $\sec^2(x - y)$ is

(a) $\frac{(n+1)^2}{2n}$

(b) $\frac{(n+1)^2}{n}$

(c) $\frac{(n+1)^2}{2}$

(d) $\frac{(n+1)^2}{4n}$

8. The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+(\cot x)^{\frac{1}{4}}}$ is equal to

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{8}$

(d) $\frac{\pi}{12}$

9. Tangents are drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$. If the chord of contact of these tangents touches the circle $x^2 + y^2 = c^2$, then a^2, b^2, c^2 are in

(a) Arithmetic Progression

(b) Geometric Progression

(c) Harmonic Progression

(d) None of these

10. Two numbers x and y are chosen at random from the set $\{1, 2, 3, 4, \ldots, 29, 30\}$. The probability that $x^2 - y^2$ is divisible by 3 is

(a) $\frac{3}{29}$

(b) $\frac{4}{29}$

(c) $\frac{5}{29}$

(d) $\frac{47}{87}$

11. Find the term independent of x in the expansion of $(\frac{3}{2}x^2 - \frac{1}{3x})^6$

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{5}{12}$

(d) $\frac{7}{12}$

12. $\cot^{-1}(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}})$ where x is neither zero nor an even multiple of π, is equal to

(a) $\frac{x}{3}$

(b) $\frac{x^2}{2}$

(c) $\frac{x^2}{3}$

(d) None of these

13. One diagonal of a square is represented by the line $\sqrt{3}x + y = 2\sqrt{3}$ intercepted by the axes. Then the co-ordinates of one of the extreme points of the other diagonal is

(a) $(1 + \sqrt{3}, \sqrt{3} - 1)$

(b) $(1 + \sqrt{3}, \sqrt{3} + 1)$

(c) $(1 - \sqrt{3}, \sqrt{3} + 1)$

(d) None of these

14. Suppose a, b, c are in Arithmetic Progression and $|a|, |b|, |c|$ each is less than 1. If

$$x = 1 + a + a^2 + a^3 + \text{.....upto infinity}$$

$$y = 1 + b + b^2 + b^3 + \text{.....upto infinity}$$

$$z = 1 + c + c^2 + c^3 + \text{.....upto infinity}$$

then

(a) x, y, z are in Arithmetic Progression

(b) x, y, z are in Geometric Progression

(c) x, y, z are in Harmonic Progression

(d) None of these

15. Determine the value of the integral $\int_1^e \frac{\ln x}{\sqrt{x}} dx$

(a) $2\sqrt{e}$

(b) $4 - 2\sqrt{e}$

(c) 4

(d) None of these

16. Find the value of $\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{\frac{1-x}{1+x}} \sin^{-1} x \, dx$

(a) $-1 + \frac{\pi}{2\sqrt{3}}$

(b) $\frac{\pi}{4}$

(c) $\frac{2\pi}{3}$

(d) $2 + \frac{\pi}{4}$

17. A project manager has to allocate a software engineer to a development task. The engineers with software development experience up to 5 years are available in the company but there is no engineer with zero experience. The relationship between the defect injection rate and experience (in years) is given as:
Defect injection rate = 3 x experience^2 + $\frac{162}{\text{experience}}$.
Identify the experience (in years) needed for the engineer to develop the software with minimum defect injection rate.

(a) 0

(b) 1

(c) 2

(d) None of these

18. If α and β are the roots of the equation $225x^2 - 15x - 2 = 0$, then $\lim_{n\to\infty} \sum_{r=1}^{n}(\alpha^r + \beta^r)$ is equal to

(a) $-\frac{19}{208}$

(b) $\frac{11}{208}$

(c) $-\frac{11}{208}$

(d) None of these

19. Let $f(x) = x(1 - |x|), |x| \leq 1$. The minimum value of f is

(a) 0

(b) -1

(c) $\frac{-1}{4}$

(d) $\frac{1}{4}$

20. Let $f(x) = \exp(g(x))$, where $g(x) = (\sin(|x|))^2$. Then

(a) g is not a continuous function

(b) g is not a bounded function

(c) g is a continuous function which is not differentiable at $x = 0$

(d) g is differentiable at all x

21. Let $x_1 = \alpha$, where $|\alpha| < 1$ is a real number. For $n = 1, 2, ...$ define

$$x_{n+1} = \begin{cases} x_n & \text{if n is odd;} \\ x_n^2 & \text{if n is even.} \end{cases}$$

What is the limit of $\{x_n\}$?

(a) 0

(b) 1

(c) -1

(d) None of these

22. The set of all real numbers x for which $\log_{2004}(\log_{2003}(\log_{2002}(\log_{2001} x)))$ is defined, is $\{x|x > c\}$. What is the value of c?

(a) 0
(b) 2001^{2002}
(c) 2002^{2003}
(d) 2003^{2004}

23. Square ABCD has area 36 and side AB is parallel to the x axis. Vertices A, B and C are on the graphs of $y = \log_a x, y = 2\log_a x$ and $y = 3\log_a x$ respectively. What is the value of a?

(a) $\sqrt[6]{3}$
(b) $\sqrt{3}$
(c) $\sqrt[3]{6}$
(d) $\sqrt{6}$

24. The smallest value of $9e^x + 25e^{-x}$, where x is a real number, is

(a) 25
(b) 30
(c) 127
(d) None of these

25. Consider the point $P(1, 1)$ on $y = x^4$, PM and PN are the two perpendiculars on the two axes, i,e., x-axis and y-axis respectively and O is the origin (0,0). Then the ratio of the two areas i.e, areas of OMP and ONP is

(a) 1 : 2
(b) 1 : 4
(c) 1 : 6
(d) None of these

26. In a factory three machines X, Y and Z produce respectively 1000, 2000 and 3000 bolts every day. The defective percentage of bolts for these three machines are $1\%, 1.5\%$ and 2% respectively. The bolts produced by the three machines get mixed and are sent through a conveyor to final assembly. Suppose a bolt is chosen at random from the bolts arrived at final assembly section and found to be defective. What is the probability that the defective bolt is produced in Machine X?

(a) 0.016

(b) 0.030

(c) 0.300

(d) None of these

27. Let R be the set of real numbers and a function $f : R \to R$ is defined as $f(x) = 2x + 1$. If $g(f(x)) = 10x + 10$, then the expression of $g(x)$, $G : R \to R$, is

(a) $x + 1$

(b) $5x$

(c) $5x + 5$

(d) None of these

28. The value of $\int_0^1 \int_0^y xye^{-x^2} dxdy$ is

(a) $\frac{e}{2}$

(b) $\frac{1}{2e}$

(c) $\frac{e}{4}$

(d) $\frac{1}{4e}$

29. Find the sum up to 8^{th} term of the following sequence
2, 5, 12, 31, 86, ... up to infinity

(a) 1121

(b) 3316

(c) 2195

(d) 3525

30. The $(m+1)th, (n+1)th, (r+1)th$ terms $(m \neq n \neq r, m, n, r > 0)$ of an Arithmetic progression are in Geometric Progression. Given that $n = \frac{2mr}{m+r}$ and for any value of n, the ratio between common difference and the first term of the Arithmetic progression is

(a) $\frac{1}{n}$

(b) $-\frac{1}{n}$

(c) $\frac{2}{n}$

(d) $-\frac{2}{n}$